Math 434 **January 25, 2010**

Notes on Sets

- A *set* X is a collection of elements (objects). Write $x \in X$ to indicate that element x is a member of set X.
- Sets X and Y are *equal* ($X = Y$) if they have the same elements:
  - For all x, $x \in X$ if and only if $x \in Y$.
- A set S is a *subset* of X (denoted by $S \subseteq X$) if all of its elements are members of X. Formally: If $s \in S$, then $s \in X$.
  - Note that from these definitions, it follows that $X \subseteq X$.
- A set S is a *proper subset* of X (denoted by $S \subset X$) if $S \subseteq X$ and $S \neq X$.
- It follows from the definitions that two sets X and Y are equal if and only if they are subsets of each other: $X = Y$ iff $Y \subseteq X$ and $X \subseteq Y$.

- The empty set (written $\varnothing$) has no elements.

- If X and Y are subsets of a set Z, then:
  - The *intersection* of X and Y is: $X \cap Y = \{z : z \in X \text{ and } z \in Y\}$
    - It follows that $X \cap Y \subseteq X$ and $X \cap Y \subseteq Y$
  - The *union* of X and Y is: $X \cup Y = \{z : z \in X \text{ or } z \in Y\}$
    - It follows that $X \subseteq X \cup Y$ and $Y \subseteq X \cup Y$
  - The *difference* between X and Y is: $X - Y = \{x : x \in X \text{ and } x \notin Y\}$
  - The *complement* of X is: $X' = Z - X = \{z : z \in Z \text{ and } z \notin X\}$
  - Sets X and Y are *disjoint* if they have no elements in common; i.e.:
    - $X \cap Y = \varnothing$
    - It follows from the definition that X and X' are disjoint: $X \cap X' = \varnothing$