# Math 112

Quantitative Reasoning

March 15, 2010

# Schedule 3/15/10

- **Reminder: Midterm next week**
  - **We're running behind on our schedule**
  - **We'll do a few minutes of game theory first**
  - **Makeup of Test 1 can also be taken next week**
    - **The most recent test counts!**
- **Voting theory**
  - **Homework questions**
  - **Classwork**
- **Manipulability of Voting Systems**
- **Weighted voting systems**

# The Three Conditions

1. All voters are treated equally. That is, if any two voters were to exchange marked ballots before submitting them, the outcome of the election would not change.
2. Both candidates are treated equally. If a new election were held and every voter were to reverse his vote, then the outcome of the election would be reversed.
3. It is monotone. If a new election were held and a single voter changed his ballot from a vote for the loser to a vote for the winner, then the outcome would not change.

# Analyze This System

- There are 3 voters (1, 2, and 3) and two candidates A and B
- The winner is whoever Voter 1 votes for
- Unless everyone else votes for that candidate also
  - In which case, the other candidate wins
- Which conditions are violated by this system?

# Analyze This System

- There are 3 voters (1, 2, and 3) and two candidates A and B
- Candidate A wins
- Unless everyone votes for candidate A
  - In which case candidate B wins
- Which conditions are violated by this system?

# The Manipulability of Voting Systems

- **A voter can vote against his actual preference in order to ensure that his candidate wins**

# Manipulability of Voting Systems
# The Chair's Paradox

- The Chair has tie-breaking power
- Preferences
  - Chair: A, B, C
  - You: B, C, A
  - Me: C, A, B
- Voting
  - You know that the chair will vote for A and I will vote for C
  - So you vote for C
  - The candidate that the chair likes least wins

# Sometimes the Three Conditions are Not Valid

- A jury trial
  - The decision must be unanimous
- Stockholders get 1 vote per share owned
- These are examples of weighted voting systems
  - In a weighted voting system, different voters have different power
  - We can compute their power using the Banzhaf power index

# Weighted Voting Systems Notation

- Assume two candidates (yes and no)
- Specify the quota (how many votes to win) and the weights of the voters
- Notation: $[q : w_1, w_3, w_3, \ldots, w_n]$
- Example: [51: 25, 26, 27]
- Example: [51: 40, 60]

# Weighted Voting Systems

- Dictatorships
- Dummy voters
- Veto power
- Power indices
  - Shapley-Shubik
  - Banzhaf
  - Counting voting combinations

# Weighted Voting Systems Calculating Power

- **Voting combination**
  - **List of voters showing how each voted**
- **Critical voter**
  - **A voter is critical if the outcome would be different if he changed his vote**
- **Banzhaf power index**
  - **A voter's power is the number of voting combinations in which he is a critical voter**

# Weighted Voting Systems Definitions

- **Winning coalition**
  - A combination of voters that results in the approval of a motion
- **Losing coalition**
  - The group of voters who voted against an approved motion
- **Blocking coalition**
  - A combination of voters that results in the defeat of a motion

# Weighted Voting Systems
# Calculating the Power Index

- **Make a list of the winning coalitions**
- **Use the *extra votes* principle to identify the critical voters in each coalition**
- **Multiply by 2 (because the number of blocking coalitions is equal to the number of winning coalitions)**

# Weighted Voting Systems
# Calculating the Power Index

- **Example: [51 : 40, 30, 20, 10]  (pg. 349)**
  - **Voters are: A: 40; B: 30; C: 20; D: 10**
- **Make a table of all winning coalitions:**
  - **[A,B,C,D]**
  - **[A,B,C]**
  - **[A,B,D]**
  - **[A,C,D]**
  - **[A,B]**
  - **[A,C]**
  - **[B,C,D]**
- **Count extra votes and find critical voters**