Math 112

Quantitative Reasoning

March 1, 2010

Schedule 3/1/10

- Reminder: no class next week
- Test 1 review
- Voting theory homework:
- Find or invent a voting rule for two alternatives that satisfies:
  - Condition 1 but neither conditions 2 nor 3
  - Condition 2 but neither 1 nor 3
  - Condition 3 but neither 1 nor 2

The Three Conditions

1. All voters are treated equally. That is, if any two voters were to exchange marked ballots before submitting them, the outcome of the election would not change.
2. Both candidates are treated equally. If a new election were held and every voter were to reverse his vote, then the outcome of the election would be reversed.
3. It is monotone. If a new election were held and a single voter changed his ballot from a vote for the loser to a vote for the winner, then the outcome would not change.

Satisfy 1 but neither 2 nor 3

1. All voters are treated equally. That is, if any two voters were to exchange marked ballots before submitting them, the outcome of the election would not change.
2. Both candidates are NOT treated equally. If a new election were held and every voter were to reverse his vote, then the outcome of the election would NOT be reversed.
3. It is NOT monotone. If a new election were held and a single voter changed his ballot from a vote for the loser to a vote for the winner, then the outcome WOULD change.

Satisfy 2 but neither 1 nor 3

1. All voters are NOT treated equally. That is, if any two voters were to exchange marked ballots before submitting them, the outcome of the election WOULD change.
2. Both candidates are treated equally. If a new election were held and every voter were to reverse his vote, then the outcome of the election would be reversed.
3. It is NOT monotone. If a new election were held and a single voter changed his ballot from a vote for the loser to a vote for the winner, then the outcome WOULD change.

Satisfy 3 but neither 1 nor 2

1. All voters are NOT treated equally. That is, if any two voters were to exchange marked ballots before submitting them, the outcome of the election WOULD change.
2. Both candidates are NOT treated equally. If a new election were held and every voter were to reverse his vote, then the outcome of the election would NOT be reversed.
3. It is monotone. If a new election were held and a single voter changed his ballot from a vote for the loser to a vote for the winner, then the outcome would not change.

Possible Ways to Vote

- Three voters: 1, 2, 3
- Two candidates: A, B
- Votes in order:
  - AAA BAA
  - AAB BAB
  - ABA BBA
  - ABB BBB

Analyze This System

- There are 3 voters (1, 2, and 3) and two candidates A and B
- If they all vote for the same candidate, A wins
- If they vote for different candidates, B wins

Analyze This System

- There are 4 voters (1, 2, 3, and 4) and two candidates A and B
- Whoever 1 votes for wins
- Unless 2, 3, and 4 vote for the same candidate, in which case, that candidate wins

Analyze This System

- There are 3 voters (1, 2, and 3) and two candidates A and B
- A wins
- Unless everyone or no one votes for B

Condorcet Winner Criterion

- **Condorcet method:**
  - **Use a preference list**
  - **Determine the winner for each one-on-one pairing of candidates**
- **For every possible sequence of preference lists, either:**
  - **There is no Condorcet winner**
  - **The voting system produces the same winner as the Condorcet system**
- **Sometimes there is no Condorcet winner**

Desirable Attributes of Voting Systems with More than Two Candidates

- **Condorcet Winner Criterion (CWC) – The winner should be the Condorcet winner if there is one.**
- **Independence of irrelevant alternatives (IIA) – It is impossible for a loser to become a winner unless at least one voter reverses the order of the loser and the previous winner.**
- **The Pareto condition – If everyone prefers candidate B to candidate A, then candidate A should not be the winner.**
- **Monotonicity – If a new election is held and the only ballot change is for some voters to move the original winner higher, then the original winner should still be a winner.**

Voting Systems with Three or More Candidates

- Plurality
- The Borda Count
- Sequential Pairwise voting with agenda
- The Hare System
- Approval Voting

Plurality

- **Only first-place votes are counted: Whoever gets more votes than anyone else is the winner.**
- **Plurality voting fails to satisfy the CWC**

The Borda Count

- **Assign points to candidates according to voter preferences**
- **The candidate with the highest voter score wins**
- **The Borda count fails to satisfy IIA**

Sequential Pairwise Voting

- **Candidates run against each other in pairs, according to a predetermined agenda**
- **Fails to satisfy the Pareto condition**

The Hare System

- **Repeatedly delete “least preferred” candidates**
- **Can produce ties**
- **Fails to satisfy monotonicity**

Approval Voting

- **Vote for as many candidates as you want**
- **This procedure is used for Secretary General of the UN**
- **Also appropriate for situations where there can be more than one winner**

The Manipulability of Voting Systems

- **A voter can vote against his actual preference in order to ensure that his candidate wins**