Combinatorics and Reasoning: Representing, Justifying, and Building Isomorphisms – Table of Contents

SECTION 1: INTRODUCTION, BACKGROUND, AND METHODOLOGY

Introduction: The historical development is described in this chapter. The partnership between the Kenilworth Public Schools and Rutgers faculty emerged as an outcome of a professional development project with the Harding K-8 Elementary School. Prior to the longitudinal study, the school-university partnership established certain working conditions, support for teachers, and the establishment of a long-term relationship with administrators, faculty, and parents. *Authors: Maher, Powell, and Uptegrove*

Chapter 1: The Longitudinal Study. We describe the study and the purpose of the research, discuss how the study began, and give the conditions under which the research was conducted. We briefly describe the mathematical ideas and ways of reasoning that emerged from the study. Details are given in later chapters. *Author: Maher*

Chapter 2: Methodology. We describe the design of the study, including selection of participants, data collection, and analysis, as well as the strand of tasks worked on by the participants. We discuss the importance of the tasks in developing ways of reasoning. *Authors: Maher and Uptegrove*

SECTION 2: FOUNDATIONS OF PROOF BUILDING (1989-1996)

Chapter 3: Representations as Tools for Building Arguments (Grades 2 and 3): We discuss how young children use representations to express their mathematical ideas while building a solution to a problem. We see how they structure the representations in response to the call of providing a justification for the problem solution. Convincing arguments emerged from work on early problems with counting schemes. *Authors: Maher and Yankelewitz*

Chapter 4: Towers: Schemes, Strategies, and Arguments (Grades 3 and 4). We discuss students' work on the towers problems, showing the emergence of different forms of reasoning (cases, contradiction, recursion, and induction). *Authors: Maher, Sran, and Yankelewitz*

Chapter 5: Building an Inductive Argument (Grade 5). Motivated by finding the sample space for a basic probability exploration, students revisit the inductive argument for building towers. *Authors: Maher, Sran, and Yankelewitz*

Chapter 6: Making Pizzas; Reasoning by Cases and by Recursion (Grade 5). We discuss how students collaboratively build representations that help them use reasoning by cases and by recursion to develop justifications for their solutions to pizza problems. *Authors: Maher, Sran, and Yankelewitz*

Chapter 7: Block Towers: From Concrete Objects to Conceptual Imagination (Grade 8). In an interview, thirteen-year-old Stephanie discusses the relationship between the towers problems and the binomial expansion, including how the towers answers can be found in Pascal's Triangle. *Author: Speiser*

SECTION 3: MAKING CONNECTIONS, EXTENDING, AND GENERALIZING (1997-2000)

Chapter 8: Responding to Ankur's Challenge: Co-construction of Argument Leading to Proof (Grade 10). We show how students' representations and arguments were refined and clarified as they were revisited. *Authors: Maher and Muter*

Chapter 9: Block Towers: Co-construction of Proof (Grade 11, November 1998). Working in groups, students found and generalized formulas, using methods including controlling for variables, justification by cases, and induction. *Authors: Tarlow and Uptegrove*

Chapter 10: Representations and Connections (Grade 10). We show how Michael's binary notation helped a group of tenth-grade students form connections among pizzas, towers, the binomial expansion, and Pascal's Triangle. *Authors: Muter and Uptegrove*

Chapter 11: Pizzas, Towers, and Binomials (Grade 11, March 1999): Representations are a source for making connections in solutions to pizza and tower problems, resulting in students mapping the structure of the solution of these problems to Pascal's Triangle. The students' increasingly sophisticated use of representations led to further development of mathematical reasoning and justification. *Author: Tarlow*

Chapter 12: Representations and Standard Notation. Student personal representations are a natural segue to introducing standard notation. The power of mapping personal and standard notations leads to opportunities to represent particular arguments in a general form, thus making possible the extension of particular ideas and ways of reasoning. We discuss how students moved from personal to standard notations in order to express in general form their understanding of solutions to the pizza and towers problems and extend their understanding in building an isomorphism to Pascal's Triangle. Further extension led to an understanding of the meaning of the addition rule for Pascal's Triangle. *Author: Uptegrove*

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