

## Chapter 1: The Longitudinal Study

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### *1.1 Theoretical View*

Where do new ideas come from? Our view is that building new ideas is a process; new ideas come from old ideas that are revisited, reviewed, extended, and connected (Davis, 1984; Maher & Davis, 1995). Building new ideas also involves the retrieval and modification of representations of existing ideas. The representations that a learner builds for a mathematical idea or procedure can take different forms – physical objects or actions on objects, words, and symbols, for example. As the learner’s experience increases, old representations become elaborated, extended, and linked to new ones (Maher, 2008; Davis & Maher, 1997).

The problem tasks that are posed to learners are critical to their learning (Francisco & Maher, 2005); they should be well defined, open-ended, and open to extension and generalization. The connections that the learner makes when analyzing and developing solutions to these problems provide further opportunity for growth in knowledge. Students are encouraged to revisit earlier problems because requirements to justify and generalize solutions can help students to see underlying mathematical structure. It is a widely accepted view that when learners understand the fundamental structure of a subject, the gap between “elementary” and “advanced” knowledge is reduced (Bruner, 1960). There is increasing evidence that learners, under certain conditions, can build meaningful, mathematical relationships and understand the structure of mathematical problems at an early age. For example, a study of Norwegian children indicated that even as young as grade 3, learners are able to unearth the underlying structure of the mathematics of problem tasks (Torkildsen, 2006).

A central component of the learning process is encouraging students to communicate their ideas. Sfard (2001) suggests that students learn to think mathematically by participating in discourse about ideas – arguing, asking questions, and anticipating feedback. We have emphasized that justifying ideas in problem solving is an essential component of mathematical reasoning (Maher, 2003, 2005, 2008; Maher & Martino, 1996; Martino & Maher, 1999). Learners, in communicating their ideas, share personal mental images – representations. When students make their representations public, they have an opportunity to talk further about them, compare them, and later revisit them. Similarities and differences in ideas naturally emerge. When learners try to convince others that their answers are correct, they can reorganize and reformulate their representations so as to make convincing arguments. In summary, students learn mathematics by engaging in the process of building their own personal representations, communicating them as ideas, and then providing support for those ideas by reorganizing and restructuring representations. Our view is that this process is a necessary prerequisite both for developing the idea of mathematical proof and for making suitable connections between problems of equivalent structure by building isomorphisms.

In this book, we discuss how a group of students developed new and increasing sophisticated mathematical ideas by revisiting, reviewing, extending, and connecting old ideas that they had begun developing in first grade. They developed and modified representations that became increasingly elaborated and extended. They participated in serious mathematical discourse. And ultimately they build a strong and durable understanding of a set of mathematical tasks. Our longitudinal work is important because it reveals the processes that these learners used to build structural understanding of mathematical tasks.

## ***1.2 Background of the Study***

The longitudinal study began in 1987 in Kenilworth, New Jersey. This was during a time when behaviorism mainly governed mathematics instruction. It was a time before the reform movement in the United States emphasizing conceptual understanding had made its entry. The K-8 Harding elementary school in the working-class community of Kenilworth, New Jersey was typical of others at that time. Half hour sessions were devoted to mathematics, and mathematics instruction was mainly rote. The rule was drill and practice for carrying out memorized procedures. For the most part, even the brightest students from the school did not excel when they moved on to high school mathematics classes, only doing average work. Most members of the community and most teachers had rather low expectations for student advancement.

But Fred Rica, principal of the Harding elementary school, had higher expectations for the students in his school. Formerly an elementary grade classroom teacher in Kenilworth, Fred Rica knew his staff and students well. Like other concerned educators, he knew when the system was not serving its student population. He turned to Rutgers University for help with instruction, first in language and literacy and then in mathematics. It was shortly after this professional development work that Fred Rica and Carolyn Maher created a partnership between the Kenilworth Public Schools and Rutgers University. It should be noted that the Rutgers-Kenilworth partnership, with its focus on students building meaning of mathematical ideas and working collaboratively with each other, began long before the National Council of Teachers of Mathematics published its reform Standards.

Initially, the project began as a teacher development intervention in mathematics. The Rutgers University team of researchers and graduate students worked for three years to help teachers build an understanding of the mathematics they were expected to teach and to learn to be attentive to the developing understanding of their students. (See O'Brien, 1994, and Davis & Maher, 1993, for a detailed study of the teacher development project.)

The project could not have survived the early years without the full support and active participation of the Kenilworth school administration. In particular, Principal Rica actively participated in the teacher training sessions, encouraged teachers to become involved, and made sure that students who were involved in the study were available to the researchers. Original financial support for the partnership came from the Kenilworth school district and through volunteer efforts of the Rutgers team. The Kenilworth school district continued to fund the study for several years as a component of its mathematics teacher development mission. The Rutgers research group received outside funding for the research from two National Science Foundation grants. The first grant awarded to Principal Investigators Robert B. Davis and Carolyn A. Maher was when the students were in grade four; the second grant awarded to Principal Investigator Carolyn A. Maher, was when students were in high school.

### **1.2.1 Teacher Development Component**

It is not surprising that the teachers at the Harding Elementary School were not prepared to teach mathematics with understanding. What is surprising was the expectation of Principal Fred Rica that the teachers were capable, with some professional development and classroom support, of understanding the mathematics they were expected to teach. In fact, this view was remarkable for its time.

The teacher development team was made up of mathematics education doctoral students who had considerable experience in schools; its first members were Alice S. Alston and Judith H. Landis. The team worked closely with Fred Rica and his teachers to establish a program of activities that involved not only videotaped teacher workshops and classroom sessions, but also study of those workshops and sessions. The Rutgers team worked directly with students and with their teachers, first observing classroom sessions and later collaborating with the teachers in the design and implementation of lessons. Alice Alston also worked in the classrooms alongside the teachers.

Principal Rica obtained school funding to support teachers' summer work to revise the existing curriculum. Two years of summer professional development assisted by John O'Brien and Alice Alston resulted in a movement from a "drill and kill" approach to one in which students' building of mathematical

understanding was central. Curriculum revisions included the use of more engaging and thoughtful lessons for the students and the introduction of manipulatives that allowed students to build models of their solutions.

Some Kenilworth teachers who participated in the teacher development programs also became involved in classroom action research. As teachers were introduced to new resources and tools, they developed new units and piloted them during the school year. Through course work opportunities at Rutgers, some teachers studied the mathematical learning of their own students (Maher, Landis & Alston, 1986; Landis and Maher, 1989; Landis, 1990; O'Brien, 1994).

### **1.2.2 Intervention Design**

The Rutgers team was interested in what mathematical concepts students could learn with minimal intervention from teachers. Classrooms were organized so that children might work together and collaborate on problem tasks. Children were encouraged to use each other as resources in their investigations, to construct models of solutions with available tools, and to revisit tasks and discuss their strategies and solutions. An important observation during the first three years was that students produced arguments that took on a variety of forms of reasoning to support their solutions to the problems. By grade four, it became increasingly clear to researchers that students' reasoning, in a natural way, took the form of proof. Children began their investigations by searching for patterns, organizing solutions, searching for completeness, deriving strategies for keeping track and checking, and then re-organizing justifications into arguments that were proof-like in structure. Using each other as resources, children freely shared ideas, questioned each other, argued about the reasonableness of ideas, and became comfortable in sharing and communicating with each other.

What encouraged both the school staff and the university collaborators was the enthusiastic feedback from students. The children enjoyed talking about their ideas; they engaged with each other with energy and enthusiasm, becoming increasingly more comfortable making their ideas public. Their way of working underscored a demand for sense making, which then evolved as a cultural norm.

This book explores student work for one of the mathematics strands of the longitudinal study: counting and combinatorics. It investigates how students' reasoning evolved over the course of the longitudinal study that continued from elementary and high school years to college.

### ***1.3 Longitudinal Study: Grades 1 through 3***

In order to study the effectiveness of the intervention, the Rutgers team decided to follow a class of students throughout their elementary grades as they worked on mathematical investigations that were not part of the school curriculum. The study began with a class of 18 first-grade students from the Harding School. These children, randomly assigned to one of three first grades, became the initial focus group; they were together for grades 1 through 3 as part of the school design. Throughout the study, students engaged in strands of thoughtful mathematics activities designed by the researchers. Although the mathematical investigations were not part of the curriculum, the concepts that were introduced would later become part of the regular school mathematics curriculum.

### ***1.4 Longitudinal Study, Grades 4 through 8***

After grade three, the students were distributed among different classrooms, according to school policy. However, the principal worked with Rutgers researchers to facilitate maintenance of a focus group of twelve students for research purposes. When families moved and new families entered the district, the composition of the focus group changed, but an attempt was made to maintain a group of comparable background and interest. Although some students stayed with the study from the start (and are still in touch

today), other students left the group and new students joined during middle school. During middle school, the group continued to meet with researchers during school hours, four to six times a year in two 90-minute sessions and one 45-minute session each time.

### ***1.5 Longitudinal Study: High School Years***

In 1996 the high school in Kenilworth was closed, as the school district became part of a regional system. The community joined forces to protest the merger and succeeded after one year. Hence, the first year of high school (ninth grade) proved disruptive for the students, although some math problem-solving sessions were conducted with small groups of students during that year in local homes, usually on Saturdays. After Kenilworth de-regionalized and the students returned to Kenilworth for the remaining three years of high school, groups of students resumed participation in the longitudinal study in informal, after-school sessions that were held during the year, usually on Friday. While students no longer met with researchers during regular class hours, fourteen students (some from the original group of first-graders and others who had joined the study at various times during middle school and high school) made time in their schedules to meet after school about four to six times a year for problem-solving sessions that lasted one to two hours or longer. This group included ten students who had been with the study since grade one, two students who had joined the study in grade six, and two who joined in high school (grade eleven).

### ***1.6 Longitudinal Study: Beyond High School***

All students in the focus group applied to Rutgers University, and all were accepted – a remarkable achievement for the district. However, not all students attended Rutgers; they attended a variety of universities, public and private; besides Rutgers, these included Kean University, St. John's University, and the University of Pennsylvania. Majors included accounting, American studies, animal science, computer science, criminal justice, economics, engineering, English, and mathematics. All are now either employed or in graduate school.

Some of the students have continued to meet occasionally with researchers during and after college. They do not generally work on problems (although sometimes old problems are revisited), but they talk about how being in the study has affected them, their academic careers, and their future plans.

In the next chapter, we detail how the study was conducted and we discuss selected problems that formed the cornerstone of the student investigations over the years.