Math 434 **January 25, 2010**

Mathematical Induction

Given statements $S(n)$, one for each natural number n, suppose that:

1. $S(1)$ is true.
2. If $S(n)$ is true, then $S(n + 1)$ is true.

Then $S(n)$ is true for all integers n.

Example

Prove that $2^n > n$ for all $n > 0$. To do this:

1. Show that $S(1)$ is true; i.e. show that $2^1 > 1$.
2. Assume $S(n)$ is true: $2^n > n$. Show that $S(n + 1)$ is true: $2^{n+1} > n + 1$.

The proof:

1. $2^1 > 1$: Yes. $2 > 1$.
2. Assume $2^n > n$.
    - Then $2 \bullet 2^n > 2n$ by the multiplication property of inequality.
    - $2 \bullet 2^n = 2^{n+1}$ by the multiplication property of exponents.
    - And $2n = n + n$ by the distributive rule.
    - $n + n \geq n + 1$ by the addition property of inequality
    - So $2n \geq n + 1$
    - Therefore, $2^{n+1} > n + 1$ by the transitive property.

Classwork

Prove: $1 + 2 + \ldots + n = ½\, n\, (n + 1)$

1. Show this is true for $n = 1$.
2. Assume that $1 + 2 + \ldots + n = ½\, n\, (n + 1)$.
   Prove that $1 + 2 + \ldots + n + (n + 1) = ½\, (n + 1)((n + 1) + 1)$.