Laws of Continuity — Calculus II review; Summer 2010

Assume that functions $f(x)$ and $g(x)$ are continuous at c. Then the following functions are also continuous at c:

- $f(x) + g(x)$
- $f(x) - g(x)$
- $k\, f(x)$ for any constant k
- $f(x) \cdot g(x)$
- $f(x) / g(x)$ if $g(x) \neq 0$

Polynomial Functions
If $P(x)$ and $Q(x)$ are polynomial functions, then:

- $P(x)$ is continuous
- $P(x) / Q(x)$ is continuous wherever $Q(x)\ g(x) \neq 0$

Other Continuous Functions
$y = \sin(x)$ and $y = \cos(x)$ are continuous
For $b > 0$, $y = b^x$ is continuous
For $b > 0$ and $b \neq 1$, $\log_b x$ is continuous for $x > 0$
If n is a natural number, then $y = x^{1/n}$ is continuous on its domain

Continuity of the Inverse Function
If $f(x)$ is a continuous function on an interval I with range R and if the inverse $f^{1}(x)$ exists, then $f^{1}(x)$ is continuous on the domain R.

Continuity of Composite Functions
Let $F(x) = f(g(x))$ be a composite function. If g is continuous at $x = c$ and f is continuous at $x = g(c)$, then $F(x)$ is continuous at $x = c$.

The Intermediate Value Theorem
If $f(x)$ is continuous on $[a,b]$ and $f(a) \neq f(b)$, then for every value M between $f(a)$ and $f(b)$, there exists at least one value c in (a,b) such that $f(c) = M$.

Existence of Zeroes
If $f(x)$ is continuous on $[a,b]$ and $f(a)$ and $f(b)$ are nonzero and have opposite signs, then $f(x)$ has a zero in (a,b).