

**Properties of Equality**

Addition If  $a = b$ , then  $a + c = b + c$

Multiplication If  $a = b$ , then  $a \cdot c = b \cdot c$

**Properties of Inequality**

Addition If  $a > b$ , then  $a + c > b + c$

If  $a < b$ , then  $a + c < b + c$

Multiplication If  $a > b$  and  $c > 0$ , then  $a \cdot c > b \cdot c$

If  $a < b$  and  $c > 0$ , then  $a \cdot c < b \cdot c$

If  $a > b$  and  $c < 0$ , then  $a \cdot c < b \cdot c$

If  $a < b$  and  $c < 0$ , then  $a \cdot c > b \cdot c$

**Properties of operations on real numbers**

Operation	Addition	Multiplication
Commutative	$a + b = b + a$	$ab = ba$
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Identity	$a + 0 = a$	$a \cdot 1 = a$
Inverse	$a + (-a) = 0$	$a \cdot 1/a = 1$ , if $a \neq 0$
Distributive Rule	$a(b + c) = ab + ac$	

**Numbers**

Natural numbers  $\mathbf{N} = \{1, 2, 3, 4, \dots\}$

Integers  $\mathbf{Z} = \{\dots -4, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational numbers  $\mathbf{Q} = \{p/q, \text{ where } p, q \in \mathbf{Z} \text{ and } q \neq 0\}$

Real numbers  $\mathbf{R} = \text{points on the number line}$

**Divisibility and prime numbers**

- Division algorithm: Given integers  $a$  and  $b$  with  $a \neq 0$ , there exist unique integers  $q$  and  $r$  such that  $b = qa + r$ , with  $0 \leq r < |a|$
- The integer  $a$  is a *divisor* of integer  $b$  if  $b = na$ , where  $n$  is an integer. Denote this by  $a|b$ .
- A natural number  $p$  is *prime* if its only divisors are 1 and  $p$ .
- Every natural number  $> 1$  is either prime or a product of primes (*composite*).