Asymptotes and Graphs Calculus II review; Summer 2010

Asymptotic behavior refers to the behavior of a function *f(x)* as either *x* or *f(x)* approaches ±∞.

A horizontal line $y = L$ is a *horizontal asymptote* if one of the following limits exists:

$$\lim_{x\to\infty} f(x) = L \qquad or \qquad \lim_{x\to-\infty} f(x) = L$$

A vertical line $x = L$ is a *vertical asymptote* if *f(x)* has an infinite limit as *x* approaches *L* from the left or right (or both):

$$\lim_{x\to L^+} f(x) = \pm\infty \qquad and/or \qquad \lim_{x\to L^-} f(x) = \pm\infty$$

Asymptotic behavior of a *rational function*: If $a_n \neq 0$ and $b_m \neq 0$, then:

$$\lim_{x\to\pm\infty}\left(\frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0}{b_m x^m + b_{m-1}x^{m-1}}\right) = \frac{a_n}{b_m}\lim_{x\to\pm\infty} x^{n-m}$$

Graphing a Function

- Determine the domain of *f*.
- Determine the signs of *f'* and *f''*.
- Note the transition points and sign combinations.
- Draw arcs of appropriate shape and asymptotic behavior.

Summary

Abbrev.	Sign Combination	Curve Type
++	$f' > 0$, $f'' > 0$	Increasing and concave up
+–	$f' > 0$, $f'' < 0$	Increasing and concave down
–+	$f' < 0$, $f'' > 0$	Decreasing and concave up
–+	$f' < 0$, $f'' < 0$	Decreasing and concave down