

Applications of the Derivative Calculus II review; summer 2010

Linear Approximation: If f is differentiable at $x = a$ and Δx is small, then:

$$\Delta f \approx f'(a)\Delta x$$

where:

$$\Delta f \approx f(a + \Delta x) - f(a). \quad \text{Equivalent form: } f(a + h) - f(a) \approx f'(a)h$$

Approximating $f(x)$ by its linearization: If x is close to a , then:

$$f(x) \approx L(x) = f'(a)(x - a) + f(a)$$

Error Bound: If $f''(x)$ is continuous, then the error E satisfies the following condition:

$$E \leq \frac{1}{2}Kh^2$$

where K is the maximum value of $|f''(x)|$ on the interval from a to $a + h$.

Extreme Values and Critical Points

Maxima and Minima: If $f(x)$ is a continuous on a closed (bounded) interval $I = [a, b]$, then $f(x)$ takes on a minimum and a maximum value in I .

Critical Point: A number c in the domain of f is called a *critical point* if either $f'(c) = 0$ or $f'(c)$ does not exist.

Fermat's Theorem: If $f(c)$ is a local min or max, then c is a critical point of f .

Extreme Value on a Closed Interval: Assume that $f(x)$ is continuous on $[a, b]$ and let $f(c)$ be the minimum or maximum value on $[a, b]$. Then c is either critical point or one of the endpoints a or b .

The Mean Value Theorem

Assume that f is continuous on $[a, b]$ and differentiable on (a, b) . Then there exists a c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem (special case of MVT): Assume that $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) . If $f(a) = f(b)$, then there exists a c in (a, b) such that $f'(c) = 0$.

Increasing/Decreasing Functions. Let f be a differentiable function on (a, b) .

- If $f'(x) > 0$ for x in (a, b) , then f is increasing on (a, b) .
- If $f'(x) < 0$ for x in (a, b) , then f is decreasing on (a, b) .

Concavity. Let f be a differentiable function on (a, b) . Then:

- f is concave up on (a, b) if $f'(x)$ is increasing on (a, b) .
- f is concave down on (a, b) if $f'(x)$ is decreasing on (a, b) .

Test for Concavity. Suppose that $f''(x)$ exists for all x in (a, b) .

- If $f''(x) > 0$ for all x in (a, b) , then f is concave up on (a, b) .
- If $f''(x) < 0$ for all x in (a, b) , then f is concave down on (a, b) .

Inflection Points: Where the concavity changes. Test for inflection points: Assume that $f''(x)$ exists for all x in (a, b) and let c be a value in (a, b) . If $f''(c) = 0$ and $f''(x)$ changes sign at $x = c$, then $f(x)$ has a point of inflection at $x = c$.

Second Derivative Test. Assume that $f(x)$ is differentiable and let c be a critical point. If $f''(x)$ exists, then:

- $f''(x) > 0$ means that $f(c)$ is a local minimum
- $f''(x) < 0$ means that $f(c)$ is a local maximum