

Math 112

Quantitative Reasoning

March 29, 2010

Schedule 3/29/10

- **Brief discussion of the midterm**
- **Questions on the homework?**
 - (p.429: 22,23)
- **Fair division continued**
- **Apportionment (if time)**

Cake Division

Proportional Procedure

- A cake-division procedure for n players is *proportional* if each player's strategy guarantees that player a piece of size at least $1/n$ of the whole, in his estimation

Fair Division Topics

- **The adjusted winner procedure**
- **The Knaster inheritance procedure**
- **Fair division and organ transplants**
- **Taking turns**
- **Divide-and-choose**
- **Cake division procedures**
 - **Proportionality**
 - **Envy**

Fair Division Definitions

- **Equitable** – A procedure is *equitable* if each player believes that he received the same fractional part of the total value
- **Envy-free** – A procedure is *envy-free* if each player has a strategy that can guarantee him or her a share that is, in his own eyes, at least as large (or desirable) as that received by any other player, no matter what other players do
- **Pareto-optimal** – A procedure is *pareto-optimal* if it produces an allocation with the property that no other allocation can make any other player better off without making some other player worse off

Adjusted Winner Procedure

1. Each party distributes 100 points over the items.
2. Each item is initially given to the party that assigned it more points. Each party then counts how many points he has received. Whoever has fewer points gets each item on which both parties placed the same number of points.
3. If the point totals are not equal, let A be the person with the higher point total and B be the other person. Start transferring items from A to B until the point totals are equal. This may involve fractional transfer of one item.
4. The order in which this is done is important. It is determined by going through the items in order of increasing point ratio. Point ratio is the fraction:

$$\frac{A's \text{ point value of the item}}{B's \text{ point value of the item}}$$

Applying the Adjusted Winner Procedure to a Merger

Point allocation		
Issue	Glaxo Wellcome	SmithKline Beecham
Name	5	10
Headquarters	25	10
Chairman	35	20
CEO	15	35
Layoffs	20	25
Total	100	100

Adjusted Winner Procedure Group Work

Point allocation in a divorce		
Issue	Hers	His
Family home		
Vacation condo		
Family SUV		
Coin collection		
Jewelry		
Stocks		
Total	100	100

Knaster Inheritance Procedure for n People

- 1. The heirs independently and simultaneously submit bids for the object.**
- 2. The high bidder is awarded the object, and he places all but $1/n$ of his money in the kitty.**
- 3. Each of the other heirs withdraws from the kitty $1/n$ of his bid.**
- 4. The money remaining in the kitty is divided among the n heirs.**

Knaster Inheritance Procedure Example

Object	Bob	Carol	Ted	Alice
House	\$120,000	\$200,000	\$140,000	\$180,000
Cabin	\$60,000	\$40,000	\$90,000	\$50,000
Boat	\$30,000	\$24,000	\$20,000	\$20,000

Knaster Inheritance Procedure

Group Work

Object	A's bid	B's bid	C's bid
Three bedroom home in Rutherford, NJ			
House furnishings – living room, dining room, kitchen, two bedrooms			
Furnished vacation home at the shore with 2 bedrooms, 1 block from the beach			
Car – 2010 Honda Acura with 10,000 miles on it and many features			

Fair Division Organ Transplant Lists

- **Waiting time – Rank according to position on list (1 = waiting longest)
Points = 10 times fractional position**
- **Suitability – 2 points for each antigen match**
- **Disadvantages – 1 point for each 10% of the population the person is sensitized to**

Organ Transplant Lists

Example

Potential Recipient	Months Waiting	Antigens Matched	Percent Sensitized
A	5	2	10
B	4.5	2	20
C	4	0	0
D	2	3	60
E	1	6	90

Points Allocated

Potential Recipient	Months Waiting	Antigens Matched	Percent Sensitized	Total Points
A	10	4	1	15
B	8	4	2	14
C	6	0	0	6
D	4	6	6	16
E	2	12	9	23

From last time:

Fair Division Definitions

- **Cake division procedure for n players:**
 - **A procedure that the players can use to allocate a cake among themselves *without an outside arbiter*, so that each player has a strategy that will guarantee that player a piece with which he is satisfied**

Cake Division

Proportional Procedure

- **Cake division for two people:**
 - **Divide-and-choose: One party divides the cake into two parts in any way he wants, and the other party chooses whichever part she wants**

Steinhaus Proportional Procedure

Lone Divider

- **Person 1 divides the cake into 3 equal pieces**
- **Persons 2 and 3 decide which piece(s) they *approve* of (they think that piece is at least $1/3$ of the whole)**
 - **If Persons 2 and 3 approve of two different pieces, the third piece goes to Person 1, and Persons 2 and 3 get the pieces they selected**
 - **If Persons 2 and 3 approve of the same piece:**
 - **Put the approved piece back together with one of the disapproved pieces and give the other disapproved piece to Person 1**
 - **Let Persons 2 and 3 use “divide and choose” on what’s left**