

Math 112

Quantitative Reasoning

April 12, 2010

Schedule 4/5/10

- **Questions on the homework?**
- **Reminder: Test 3 is next week! Topics:**
 - **Mostly new stuff: Weighted voting, fair division, game theory**
 - **A few old things: Circuits, trees, digraphs**
- **Game theory**
 - **The pitcher/catcher duel**
 - **Revisiting two-person partial conflict games**
 - **Nash equilibrium**
 - **A truel**

Game Theory – Review Definitions

- **Players – Participants in a game**
 - May be people, countries, organizations
- **Strategies – Options available to the players in a game**
- **Outcomes – Consequences of players' strategies**
- **Preferences – Players prefer some outcomes to others**
 - They want to win!
- **Rational choice – Players select strategies in order to maximize the chance of a preferred outcome**

Two-Player Total Conflict Game

- **When one player wins, the other loses**
 - There is no incentive for cooperation
- **There is a straightforward way to determine the best strategy**
 - Payoff table with minimax/maximin strategy
- **The value of the game is the best outcome that both players can guarantee. If a game has a saddlepoint, that's the value.**

A Duel between Pitcher and Batter

- The pitcher can throw either a fastball (F) or a slow curve (C)
- The batter must guess which pitch is coming
- Both players know the batter's batting average:
 - 0.300 if the batter guesses F and the pitcher throws F
 - 0.200 if the batter guesses F and the pitcher throws C
 - 0.100 if the batter guesses C and the pitcher throws F
 - 0.500 if the batter guesses C and the pitcher throws C
- What are the best strategies for the pitcher and batter?

Classwork

- **Use algebra to work out the strategies for a pitcher and a catcher in the situation described**
- **The pitcher should decide the relative fraction of time to pitch a fastball or a curve**
- **The catcher should decide the relative fraction of time to expect a fastball or a curve**

More Definitions

- **Pure strategy** – Each of the definite courses of action a player can take is called a pure strategy.
- **Mixed strategy** – A strategy in which the course of action is randomly chosen from one of the pure strategies in the following way. Each pure strategy is assigned some probability, indicating the relative frequency with which the pure strategy will be played. The specific strategy used at any given time is selected by the use of a randomizing device.
- **Expected value:** See equation on page 476
- **Fair game:** Has a value of zero

Strategy Definitions

- **Minimax theory**
- **Variable sum games**
- **Nash equilibrium – when no player can benefit by departing unilaterally from his strategy, the strategies of the players constitute a Nash equilibrium**
- **In Prisoner's Dilemma, the strategy "defect" dominates "cooperate" for both players, even though the "both defect" outcome is worst for both players than the "both cooperate" outcome**

Prisoner's Dilemma

- **Last week, each group played two-person partial conflict games and reported on the results:**
 - **Prisoner's dilemma**
 - Cooperate or defect?
 - **The arms race**
 - Arm or disarm?
 - **Chicken**
 - Swerve or don't swerve?

A Truel (Duel with 3 People)

- **Each player can either fire or not fire his gun at either of the other two players. All players have one bullet. All are perfect shots (i.e. they never miss).**
- **The goal of each player is:**
 - **Survive**
 - **Get rid of as many other players as possible**
- **Figure this out now: What will players do if play is:**
 - **Simultaneous?**
 - **Sequential?**

Identification Numbers

- **ISBN**
- **Bank ID numbers**
- **Bar codes**