

# **Math 112**

Quantitative Reasoning

April 5, 2010

# **Schedule 4/5/10**

- **Questions on the homework?**
- **Fair division continued**
- **Game theory**

# Fair Division Topics

- **Two weeks ago**
  - Cake division procedures, including divide-and-choose and the Steinhaus proportional procedure
- **Last week**
  - The adjusted winner procedure
  - The Knaster inheritance procedure
- **Omit these topics:**
  - Fair division and organ transplants
  - Taking turns
  - Envy
- **This week**
  - Practice the Steinhaus proportional procedure and the Knaster inheritance procedure

# **Knaster Inheritance Procedure**

## **More Group Work**

<b>Object</b>	<b>A's bid</b>	<b>B's bid</b>	<b>C's bid</b>
<b>Dad's "almost antique" car collection – 1950 Studebaker pickup truck, 1960 Ford Thunderbird, and 1958 Ford Edsel</b>			
<b>Mom's jewelry – 1 carat diamond ring, cultured pearl necklace, 14 carat gold necklace and bracelet set, 4 pins made with semi-precious stones</b>			
<b>Furnished 3-bedroom vacation home in the Poconos, close to skiing, hiking, and golfing, and with a large swimming pool</b>			
<b>Brand new 2010 Lexus SUV with many features</b>			

# Steinhaus Proportional Procedure

## Lone Divider

- **Person 1 divides the cake into 3 equal pieces**
- **Persons 2 and 3 decide which piece(s) they *approve* of (they think that piece is at least  $1/3$  of the whole)**
  - **If Persons 2 and 3 approve of two different pieces, the third piece goes to Person 1, and Persons 2 and 3 get the pieces they selected**
  - **If Persons 2 and 3 approve of the same piece:**
    - **Put the approved piece back together with one of the disapproved pieces and give the other disapproved piece to Person 1**
    - **Let Persons 2 and 3 use “divide and choose” on what’s left**

# Prisoner's Dilemma

- **Each group will play a version of a two-person game of partial conflict and report on the results:**
  - **Prisoner's dilemma**
  - **The arms race**
  - **Chicken**

# Game Theory – Definitions

- **Players – Participants in a game**
  - May be people, countries, organizations
- **Strategies – Options available to the players in a game**
- **Outcomes – Consequences of players' strategies**
- **Preferences – Players prefer some outcomes to others**
  - They want to win!
- **Rational choice – Players select strategies in order to maximize the chance of a preferred outcome**

# **Two-Player Total Conflict Game**

- **When one player wins, the other loses**
  - **There is no incentive for cooperation**
- **There is a straightforward way to determine the best strategy**

# The Location Game

- **Page 469:** Two partners, Henry and Lisa, plan to locate a new restaurant at a busy intersection. They agree on all aspects of the restaurant except one. Lisa likes low elevations and Henry likes high elevations.
- The layout for their location is shown in Figure 15-1 on page 470. Table 15-1 shows the altitudes of the nine possible locations for the restaurant.
- Henry and Lisa decide to turn their disagreement into a game: Henry will select one of the three routes A, B, or C. At the same time, Lisa will select one of the three highways 1, 2, or 3. Because they will make their choices at the same time, neither one can predict ahead of time what the other will do.

# The Location Game

- **Henry's strategy:**
  - For each choice of route, consider the worst-case scenario (lowest elevation)
  - Pick the route with the *best* worst-case scenario
- **Lisa's strategy:**
  - For each choice of highway, consider the worst-case scenario (highest elevation)
  - Pick the highway with the *best* worst-case scenario

# Definitions

- **Maximin:** The maximum value of the minimum numbers in the rows of a table
- **Minimax:** The minimum value of the maximum numbers in the columns of a table
- **Saddlepoint:** When a row minimum and a column maximum are the same
- **Value:** The best outcome that both players can guarantee
- **Important point:** There is no need for secrecy in a game with a saddlepoint. Why?

# The Restricted Location Game

- Henry and Lisa play the same game as before, but this time, Highway 2 and Route B are not possible choices
- There is no saddlepoint
- How do the minimax and maximin strategies work out now?